Formulating a dissimilarity metric for comparison of driving scenarios for Automated Driving Systems

Bharath Bhaskar Mahadikar[1], Nishant Rajesh[1,2], Kevin Tom Kurian[1], Erjen Lefeber[1], Jeroen Ploeg[2], Nathan van de Wouw[1], Mohsen Alirezaei[1,2]

***Abstract*— Safety assessment is one of the main challenges in deploying Automated Driving Systems (ADSs) on public roads. Scenario-based assessment is a common method to test such systems. Such scenario-based testing involves modeling the ADSs in a simulation environment to examine and evaluate their safety. Due to the complexity and uncertainty of the driving environment, the number of possible scenarios that ADSs can encounter is virtually infinite and there is a need for reduction of possible scenarios to a finite set. This research presents a generic framework to formulate a dissimilarity metric, which focuses on the comparison of driving scenarios on their most critical scenes, to reduce the number of possible scenarios into a finite and computationally manageable set.**

I. Introduction

Autonomous vehicles are undergoing intensive research and development, with plans for deployment on public roads. Amidst this progress, a significant challenge faced by the automotive industry revolves around the verification and validation (V&V) of Automated Driving Systems (ADSs). As automation levels increase, the traditional approach of evaluating vehicles through extensive real-world driving use cases becomes impractical and insufficient for ADSs assessment [1]. With the system taking on more driving functions and reducing driver dependency, it becomes crucial to gauge the system's intelligence in responding to diverse driving scenarios. These scenarios include a variety of possibilities, such as traffic congestion, accidents, and vehicle cut-ins.

ADSs are complex systems involving various sub-systems working together to improve overall safety, optimize energy usage, and prevent traffic congestion through connectivity and automation. A minimum number of expressive tests should be selected to verify these functionalities. These expressive tests should contain diverse challenging scenarios, thoroughly assessing the system's capabilities and responses in real-world-like conditions. One way to run these expressive tests is by scenario-based testing.

Due to the complexity and uncertainty of the driving environment, the number of such scenarios that ADSs may encounter is virtually infinite [2]. The International Organization for Standardization (ISO) introduced the ISO/PAS 21448 [3], which focuses on safety-relevant hazards that a vehicle may induce. These specifications introduce Safety of the Intended Functionality (SOTIF), which focuses on scenario-based testing for safety assurance.

Many researchers have explored techniques to generate and categorize scenarios automatically to find such safety-critical scenarios [4]–[7]. However, to reduce the number of scenarios to be tested and to maximize Operational Design Domain (ODD) coverage, it is necessary to select scenarios that evaluate ADSs performance in diverse conditions. To this end, there is a need for a quantitative dissimilarity metric to compare scenarios, which can further be used to quantify scenario diversity and eliminate redundancies.

To formulate a quantitative measure for assessing the similarity/dissimilarity between two driving scenarios, it is first important to define the notion of a driving scenario. Ulbrich et al. [8] defined a scene, a scenario, and relevant terminologies in their research. A *scene* is a snapshot of the environment at any given instant and a *scenario* describes the temporal development in a sequence of scenes. These interpretations are adopted by most of the studies that follow.

Next, it is crucial to understand what is characterized as similar/dissimilar between driving scenarios. As a human observer, one can consider multiple *features* to call a set of driving scenarios similar. These features could include (but are not limited to) the position, orientation, speed, and acceleration of vehicles. At this stage, given any two driving scenarios, the choice for characterizing them as similar becomes a subjective matter based on the features considered. In addition, including as many features as possible increases the problem's complexity significantly.

To address this challenge, researchers investigated general dissimilarity/trajectory metrics that differentiate between scenarios based on a set of features. Su et al. [9], Sousa et al. [10], and Tao et al. [11] present surveys on existing measures of similarity in literature. The most commonly used measures depending on the application are Euclidean distance, Lock-step Euclidean distance, and Dynamic Time Warping.

Apart from the abovementioned measures, a few researchers propose new measures that aim at scenario dissimilarity. Kerber et al. [12] introduce an approach for scenario clustering through spatiotemporal filtering of driving scenarios within an available driving database. Bernard et al. [13] presented a trajectory clustering approach for scenario-based testing of ADSs. The concept of scenario dissimilarity in novelty assessment is introduced in [14]. The author first represents a scene by each vehicle's position and orienta-

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[1]Eindhoven University of Technology

[2]Siemens Digital Industries Software B.V.

tion, distance, and two angles to address relative positions and orientations. The scenario dissimilarity is obtained by computing the average distance between the corresponding scenes of each scenario using the Euclidean distance.

The primary emphasis of the approaches by [12]–[14] is on computing dissimilarity based on a scenario trajectory-level formulation. From a safety standpoint, the most crucial information in a scenario is found near the most critical scenes. Hence, a key drawback in existing methods is relying on the full trajectory to calculate dissimilarity. The development of the scenario trajectory, while important, is not as relevant as the safety-critical incidents for the selection of scenarios that provide diverse challenges for the ADSs. Consider a scenario: a vehicle approaching an intersection, encountering a previously occluded pedestrian. Safety crucially depends on the vehicle's and pedestrian's proximity, emphasizing their positions and velocities. However, present methods analyze entire trajectories, instead of safety's focal points.

To address this issue, we suggest a novel approach to compute dissimilarity, focused specifically on the most safety-relevant scene. To the best of the authors' knowledge, no current methods specifically concentrate on formulating a generic dissimilarity metric based on the most safety-relevant scene. In this research, we propose a method for computing scenario dissimilarity which prioritizes the most safety-critical scene, while accounting for the complete trajectory information of involved actors.

II. Metric Foundation

This section illustrates the methodology followed to model a dynamical system and define a scenario as a trajectory of such a system. It further discusses the concept of dissimilarity and the factors to consider for formulating a measure to differentiate between driving scenarios.

A. Dissimilarity Metric

Let a concrete scenario be mathematically defined as a temporal trajectory of certain Variables of Interest (VOIs), denoted as $\zeta \in \mathfrak{B}$, where $\mathfrak{B}$ is the set of allowable concrete scenarios. The dissimilarity indicator then is a function

$$\mathfrak{d}(\zeta_1, \zeta_2) : \mathfrak{B} \times \mathfrak{B} \rightarrow \mathbb{R}_0^+, \quad (1)$$

where ζ_1, ζ_2 are two concrete scenarios in $\mathfrak{B}$[1]. Depending on the application and requirements, the complete trajectories of the VOIs, or their values at certain relevant time instants could be used for computing the dissimilarity between scenarios. Additionally, for easier interpretation, we scale the values of $\mathfrak{d}$ to lie in the closed interval $[0, 1]$, with low values representing lower values of dissimilarity. Given this general definition of dissimilarity, the following subsections aim to develop a dissimilarity metric, illustrated by an example system.

[1]In general, the dissimilarity indicator may or may not satisfy all the properties of a mathematical metric, which include non-negativity, symmetry, triangle inequality, and separation [15].

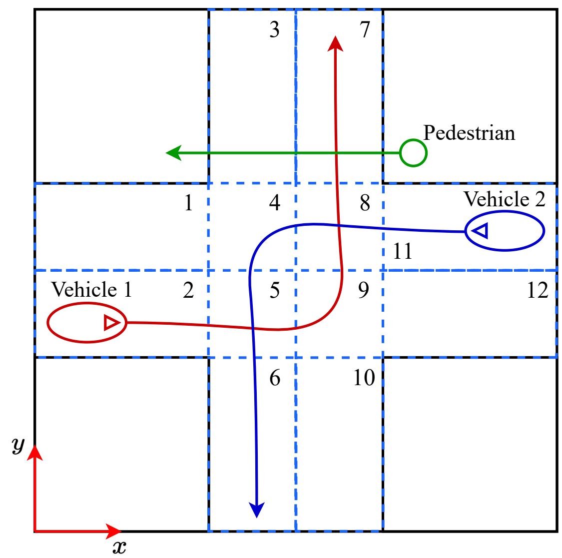

Fig. 1. Modeled System of Interest (SOI) with scenario trajectories for all actors and division of the SOI into grid cells (mesh).

B. Modelling Traffic Systems

To model a traffic system incorporating driving scenarios, fundamental parameters such as the road configuration, number of actors (vehicles, pedestrians, scenery elements), and a region boundary must be selected. For this work, an intersection road layout has been modeled as an example. The terms required to define such a system are listed below:

1) Scenery: The boundary box considered is a square with 50 m side length as shown in Fig. 1. The width of each lane is 3.75 m based on standard road dimensions.
2) Actors: The vehicles and the pedestrians, modeled as ellipses and circles, respectively, to have an approximate mathematical representation in 2D, represent the actors.
3) Variables of Interest (VOIs): These include the positions, orientations, and velocities of the actors within the boundary box. They are denoted by $(x_\text{v}, y_\text{v}, \theta_\text{v})_\text{i}$, $(v_\text{xv}, v_\text{yv})_\text{i}$ for vehicle i (where i $\in$ $\{1, 2\}$), and $(x_\text{p}, y_\text{p}, \theta_\text{p})$, $(v_\text{xp}, v_\text{yp})$ for the pedestrian.
4) Constraints and assumptions:
    a) The actors are constrained to move within the boundary box.
        i) The vehicles are modeled to move only along the road and not anywhere else in the scenery.
        ii) The pedestrian is free to move anywhere within the boundary box.
    b) The actors, modeled as ellipses and circles, cannot overlap at any time in the scenario.

A key point to note at this stage is that this system is just an example to understand what could be considered similar between two scenario trajectories. The approach could be extended to other traffic systems as well.

In the example considered, the features are limited to the positions and orientation, which are derived based on the velocity of the actors. To differentiate between two scenarios within the modeled system, computing a difference between position and orientation should fully represent the dissimilarity.

C. Abstraction with a Mesh grid

The dissimilarity metric focuses mainly on the scene posing the greatest safety risk. However, it is important not to disregard the trajectory information leading to this critical scene. To address differences in actor trajectories, the system is first abstracted into a mesh grid, dividing it into distinct areas. Comparing the sequence of visited grid cells offers a high-level understanding of actor motion, enabling swift scenario comparisons to determine if further detailed analysis is necessary. If actor motion within the mesh grid appears similar across scenarios, a closer comparison of the safety-critical scene can be performed.

A sample method for meshing the modeled System of Interest (SOI) is shown in Fig. 1, thus dividing the road network into twelve grid cells. This meshing of the scenario space adds the first level of dissimilarity: Two scenarios where the actor's trajectories lay on different grid cells can already be considered distinct. There can be multiple ways in which the road network in the modeled SOI can be divided into meshes. Selecting a particular mesh grid is application-specific, like the choice of features for evaluating dissimilarity. The mesh grid selected will therefore be unique to the designer. This paper provides one possible set of guidelines to the designer to choose a mesh grid for any road network in an SOI. These guidelines are:

1) The width of each grid cell is equal to the lane width under consideration.
2) Each lane is one grid cell until:
   a) The end of the lane, as per lane markings.
   b) The lane splits into two to more lanes, where each split lane is a new grid cell.
   c) The lane joins a second lane or a junction. For example, lanes merging into a highway or intersection.
   d) A pedestrian/bicycle crossing splits a lane. In this case, it divides the lane into three grid cells, including the crossing.
3) The presence of objects in scenery (such as traffic lights and signboards) splits a lane into two grid cells.

In formulating these guidelines, a mindful approach is taken to accommodate the potential inclusion of real-world road layouts, lane markings, and scenery objects.

D. Dissimilarity at a Scene

Scenarios represent variable trajectories over time, where the entire trajectory holds significance. However, the critical behavior of the ADS arises from the combination of the various external factors at a certain instant. This particular instant in the scenario is crucial for evaluating ADS performance. Therefore, to extract safety-relevant information from a scenario, we propose to compute the dissimilarity based on the VOIs at that particular instant.

From a broad perspective, the utmost critical scenario arises from the combination of collision proximity and severity. This combination of collision proximity and severity can be measured in terms of a criticality metric, which can then be used to identify the most critical scene in the scenario. Thus, in this study, the dissimilarity between any two scenarios is evaluated based on differences at the most critical scenes of each scenario. This simplification for the dissimilarity assessment reduces the problem's domain from trajectory analysis to point-level analysis.

It should be noted here that criticality and dissimilarity are different. It is also important to note that the choice of the criticality metric, and the features for computing dissimilarity heavily influence the values of dissimilarity. For example, while comparing scenarios for testing a motion planning system, important features could be the locations and velocities of all actors at the instant of criticality. In contrast, if the goal is to evaluate the perception system, features such as vehicle color or size might also be important. Based on the requirements of the application, the criticality metric and the features for dissimilarity metric computation can be chosen in a manner that incorporates all the relevant information for the comparison of scenarios for the specific application.

E. Pre-conditions for the dissimilarity metric

Based on the work consolidated so far, considering the modeling approach and the definition of the mesh grid, the key hypotheses and pre-conditions for a dissimilarity metric are outlined below:

1) Two scenarios can ***only*** be compared if they lie within the same $\mathfrak{B}$. Scenarios in a different $\mathfrak{B}$ are already considered 100% dissimilar.
2) Two scenarios can ***only*** be compared if all the actors' trajectories lay on the same sequence of grid cells. If the sequence of grid cells differs between two scenarios, they are considered 100% dissimilar.
3) The difference between the two scenarios will be evaluated by comparing features at the most critical scene in each scenario and not the entire scenario.

These pre-conditions define the boundaries for the dissimilarity metric definition. Building upon these fundamental concepts, the subsequent section explains how the dissimilarity metric calculation is performed.

III. METRIC FORMULATION

To evaluate the dissimilarity $\mathfrak{d}$ between two driving scenarios, the difference in the VOIs is to be evaluated at the most critical scene of the scenario. In this study, the minimum distance between actors across the entire scenario serves as the measure of criticality. Focusing on minimum distance is key for criticality assessment as it directly signifies imminent collision risk. The VOIs at the instant of maximum criticality are chosen to calculate dissimilarity. These variables form the *features* used for comparison of scenarios and are represented by the position $(x_\mathrm{i}, y_\mathrm{i})$ and orientation (θ_i) of the actors in the global frame of reference, where $\mathrm{i} \in \{\mathrm{v}, \mathrm{p}\}$.

This section introduces two different types of *features* to compute the dissimilarity, namely *discrete* and *continuous*, based on the nature of the variables used to compute them. *Discrete* features have distinct, unordered values and can be categorized into groups. An example of a discrete feature could be the actor type which collides with the ego vehicle. These features, if different between scenarios, lead to completely different scenarios. *Continuous* features represent an infinite number of values within a defined range. These features represent the precision of any variable being measured, for example, the values of positions or velocities of actors at the instant of maximum criticality.

A. Discrete and Continuous Features

Consider two scenarios, represented by these VOIs $(x_\mathrm{v}, y_\mathrm{v}, \theta_\mathrm{v})_\mathrm{i}$ where $\mathrm{i} \in \{1, 2\}$, and $(x_\mathrm{p}, y_\mathrm{p}, \theta_\mathrm{p})$, as inputs to compute the dissimilarity. Based on the positions returned at the event of maximum criticality, the minimum distance can occur between only two actors in the scenario. In this case, it can be with the second vehicle or the pedestrian. This understanding introduces the first *discrete feature*: Actor type at maximum criticality denoted by $\mathrm{AT_c}$. This feature belongs to a finite set of all combinations of actor interactions concerning the test (ego) vehicle. This finite set for the system considered is then: {'Vehicle-Vehicle', 'Vehicle-Pedestrian'}. The difference between $\mathrm{AT_c}$ of two scenarios is then calculated as:

$$\Delta\mathrm{AT_c} = \begin{cases} 0 & \text{if } (\mathrm{AT_c})_1 == (\mathrm{AT_c})_2 \\ 1 & \text{otherwise.} \end{cases} \tag{2}$$

Introducing the discrete features helps in adding a level of abstraction. Logically, two scenarios that result in $\Delta\mathrm{AT_c}$ returning 1 from (2) indicate dissimilarity $\mathfrak{d} = 1$. Both scenarios are hence important for testing and need to be selected. If $\Delta\mathrm{AT_c}$ returns 0, further analysis is needed for dissimilarity.

Another discrete feature, based on the location of the test vehicle in the mesh grid, is also introduced: Critical grid cell number denoted by $\mathrm{GC_c}$. The $\mathrm{GC_c}$ for each scenario is chosen based on the point of the maximum criticality on the ego vehicle. The difference in the grid cell number between the two scenarios, at maximum criticality, is calculated as:

$$\Delta\mathrm{GC_c} = \begin{cases} 0 & \text{if } (\mathrm{GC_c})_1 == (\mathrm{GC_c})_2 \\ 1 & \text{otherwise.} \end{cases} \tag{3}$$

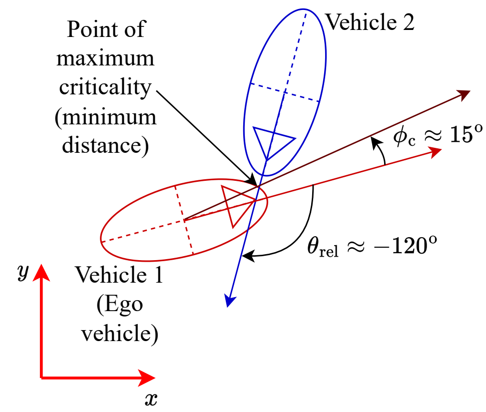

Fig. 2. Representation of Relative heading θ_rel and angles at maximum criticality in the local frame of reference ϕ_c. Counter-clockwise measurement is considered positive, and clockwise measurement is considered negative.

This discrete feature implicitly considers the differences in the positions of the actors: $(x_\mathrm{v}, y_\mathrm{v})_1$ with respect to $(x_\mathrm{v}, y_\mathrm{v})_2$ or $(x_\mathrm{p}, y_\mathrm{p})$, based on the grid cell where they lie. Similar to the actor type at minimum distance, (3) returns 1 if the grid cell of the maximum criticality point with respect to the vehicle-under-test between the two scenarios is different, thus reflecting dissimilarity $\mathfrak{d} = 1$. If $\Delta\mathrm{GC_c}$ returns 0, further analysis is needed for dissimilarity. Based on the desired application and the intent of the designer for the dissimilarity metric, more discrete features can be added at this stage.

Next, the notion of *continuous features* is introduced for the angles obtained at maximum criticality. Two angles are derived at the point of minimum distance: the relative heading angle θ_rel and the potential collision angle ϕ_c in the test vehicle frame. The physical representation of the two angles in a critical scene is shown in Fig. 2. These angles are pivotal in differentiating between critical scenarios due to their influence on the outcome. The θ_rel defines the orientation difference between two actors, which determines how they approach each other. The ϕ_c indicates the location of the other vehicle in the ego-vehicle frame and provides insight into the potential severity of collision as well as the responsibility of the ego vehicle in the incident.

Multiple methods exist for computing the difference between angles. One possible way to achieve this is using the cosine similarity index [16]. The cosine similarity returns values in the range $[0, 2]$, which can be scaled to $[0, 1]$ by dividing the result by two. The cosine difference of the angles θ_rel and ϕ_c between two scenarios, denoted by $\Delta\theta$ and $\Delta\phi$ respectively, is then calculated as:

$$\Delta\theta = [1 - \cos(\theta_\mathrm{rel1} - \theta_\mathrm{rel2})]/2 \tag{4a}$$

$$\Delta\phi = [1 - \cos(\phi_\mathrm{c1} - \phi_\mathrm{c2})]/2. \tag{4b}$$

It is important to note that using cosine similarity is only one of the possible choices to compute dissimilarity. Using other comparison metrics will result in different values.

B. Dissimilarity Calculation

Having introduced two discrete and two continuous features, the next step involves mapping them into a single value representing the dissimilarity. The mapping is done at two levels because comparing discrete features leads to values of either 0 or 1, and while comparing continuous features leads to values between 0 and 1. It is important to note that the continuous features are ***only*** considered for dissimilarity ***if*** the discrete features are the same between scenarios. This step aids in a quick estimation of the dissimilarity with higher priority given to *discrete features* over *continuous features*.

The continuous features must first be mapped into a single value, and then they can combined with the discrete features to calculate the value of dissimilarity. In this research, the continuous features $\Delta\theta$ and $\Delta\phi$ are combined to a single value σ_{c} by averaging, as:

$$\sigma_{\text{c}} = (\Delta\theta + \Delta\phi)/2. \tag{5}$$

Once the continuous features are combined into σ_{c}, the next step is to combine them with the discrete features while tending to the requirement of considering the combined continuous value ***only if*** all discrete features hold the value 0. However, multiple methods exist to satisfy this requirement, and the authors wish to highlight one approach. A method that satisfies the requirement for top-level mapping is the *maximum* function, applied over all the discrete and the combined continuous features. It is given by:

$$\mathfrak{d}(\text{Scenario1}, \text{Scenario2}) = \max(\Delta\text{AT}_{\text{c}}, \Delta\text{GC}_{\text{c}}, \sigma_{\text{c}}). \tag{6}$$

The final result from the *maximum* function thus represents the dissimilarity between a set of scenarios. Implicitly considered is the highest level of dissimilarity, where the same sequence of mesh grid cells is essential for comparison; otherwise, it results in $\mathfrak{d} = 1$.

IV. Metric Implementation

To showcase the formulated dissimilarity metric and evaluate its effectiveness, an exemplary computation of dissimilarity using sample scenario trajectories is done within the modeled system and discussed in this section. Additionally, the usage of this metric to categorize scenarios using a clustering algorithm is also performed and analyzed.

A. Dissimilarity for the modeled system

In the modeled example (see Section II-B), with the intersection type of road junction, an example of the possible paths for the three actors is depicted in Fig. 1. The ego vehicle starts its trajectory from the left side of the intersection, followed by executing a left turn. The grid cell sequence that forms the vehicle path 1 is $(2, 5, 9, 8, 7)$. The second vehicle (vehicle 2) also makes a left turn, but it initiates the maneuver from the opposite lane and the opposing side of the ego-vehicle. For vehicle 2, the grid cell sequence of the path is $(11, 8, 4, 5, 6)$. Additionally, the pedestrian's simulated movement involves crossing the road and passing through grid cells 7 and 3. The initial velocities of the actors are defined as parameters, which can be varied to obtain different concrete scenarios. It is assumed that the actors move with a constant speed along the defined path throughout the entire scenario.

Different initial velocities for the actors within the considered grid cell sequence can result in different collision types. The different collision types can occur in grid cells $\{2, 5, 7, 8, 9\}$ (the ego vehicle drives through these cells) with varying collision angles. As per the simplifications made in Section III, the scenarios with these collision modes can be depicted with two discrete and two continuous variables: AT_{c}, GC_{c}, θ_{rel} and ϕ_{c}. Using the dissimilarity metric proposed, for example, between 2 collision modes with the same AT_{c} and GC_{c}, and different continuous variables $(\theta_{\text{rel}}, \phi_{\text{c}}) = (-90, 40)^{\circ}$ and $(\theta_{\text{rel}}, \phi_{\text{c}}) = (-90, -30)^{\circ}$, we get $\Delta\theta = 0$ and $\Delta\phi = 0.3289$ from (4). Then the combined value of dissimilarity is computed as 0.1645 from (6).

The example considered represents a frontal region of the minimum distance for the test vehicle, indicating a possibility of a frontal collision. It can be realized with the angle ϕ_{c} having values of 40 and -30 between scenarios, which indicates the point of maximum criticality being on either side of the ego vehicle. Therefore, the calculation of dissimilarity with a large difference in ϕ_{c} still leads to a low value of 0.1645, thereby representing a small value of dissimilarity between scenarios with similar maximum criticality conditions.

It is important to now evaluate if the formulated dissimilarity indicator satisfies the properties of a metric. As explained in [17], the properties of a metric are:

1) Non-negativity: The dissimilarity metric is formulated to lie within the $[0, 1]$ bounds. Consequently, the dissimilarity between any two scenarios will be a non-negative real number, thereby satisfying this axiom.
2) Identity of indiscernibles: This axiom is satisfied when the dissimilarity between two scenarios is zero, if and only if both scenarios are exactly the same. Here, the dissimilarity $\mathfrak{d}$ between the two scenarios will be 0 if:

$$\begin{aligned} &(\text{AT}_{\text{c}})_1 = (\text{AT}_{\text{c}})_2,\ (\text{GC}_{\text{c}})_1 = (\text{GC}_{\text{c}})_2 \\ &\theta_{\text{rel1}} = \theta_{\text{rel2}},\ \phi_{\text{c1}} = \phi_{\text{c2}} \end{aligned} \tag{7}$$

In contrast, dissimilarity $\mathfrak{d} = 0$ implies:

$$\Delta\text{AT}_{\text{c}} = 0,\ \Delta\text{GC}_{\text{c}} = 0,\ \Delta\theta = 0,\ \Delta\phi = 0 \tag{8}$$

Variables $\Delta\text{AT}_{\text{c}}$ and $\Delta\text{GC}_{\text{c}}$ can only result in 0 if the first two conditions in (7) are satisfied. On the other hand, the variables $\Delta\theta$ and $\Delta\phi$ can be 0 when:

$$\begin{aligned} &\theta_{\text{rel1}} = \theta_{\text{rel2}} \text{ OR } \theta_{\text{rel1}} = \theta_{\text{rel2}} \pm 2\pi \\ &\phi_{\text{c1}} = \phi_{\text{c2}} \text{ OR } \phi_{\text{c1}} = \phi_{\text{c2}} \pm 2\pi \end{aligned} \tag{9}$$

Since the angles θ_{rel} and ϕ_{c} belong to the range $[-\pi, \pi]$, $\Delta\theta$ and $\Delta\phi$ are equal to 0 at the limits of the angle range. Therefore, the identity axiom is satisfied for the current formulation of dissimilarity.

3) Symmetry: This axiom is satisfied when the dissimilarity between two scenarios fulfills the following:

$$\mathfrak{d}(\zeta_1, \zeta_2) = \mathfrak{d}(\zeta_2, \zeta_1) \tag{10}$$

The cosine similarity formulation in $\Delta\theta$ and $\Delta\phi$ for dissimilarity ensures that this axiom is satisfied.

4) Triangle Property: To analyze if the triangle property is satisfied, consider three scenarios ζ_1, ζ_2, and ζ_3 having a collision between the two vehicles at grid cell 5 from Fig. 1. Let the continuous variables for these scenarios have the following values:

$$\begin{aligned} \zeta_1 &: (\theta_{\mathrm{rel}}, \phi_{\mathrm{c}}) = (-90°, 0°) \\ \zeta_2 &: (\theta_{\mathrm{rel}}, \phi_{\mathrm{c}}) = (-90°, 45°) \\ \zeta_3 &: (\theta_{\mathrm{rel}}, \phi_{\mathrm{c}}) = (-90°, 90°) \end{aligned} \tag{11}$$

Applying the dissimilarity metric to these scenarios results in $\mathfrak{d}(\zeta_1, \zeta_2) = 0.0732$, $\mathfrak{d}(\zeta_2, \zeta_3) = 0.0732$, and $\mathfrak{d}(\zeta_1, \zeta_3) = 0.25$ This implies that:

$$\mathfrak{d}(\zeta_1, \zeta_2) + \mathfrak{d}(\zeta_2, \zeta_3) \ngeq \mathfrak{d}(\zeta_1, \zeta_3) \tag{12}$$

Therefore, the formulated dissimilarity metric does not satisfy the triangle property.

The type of metric satisfying the first three axioms and not satisfying the triangle property is a Semimetric [17]. The formulated dissimilarity indicator is therefore a Semimetric.

B. Clustering scenarios using dissimilarity

The purpose of the dissimilarity metric is to ensure diversity by preventing redundant scenarios. In this sub-section, an example is provided to demonstrate the application of the dissimilarity semimetric for clustering a set of scenarios using a clustering algorithm. The interaction of the ego vehicle with the pedestrian is completely different from the second vehicle and hence results in the dissimilarity of 1 due to $\Delta\mathrm{AT}_{\mathrm{c}} = 1$. For simplicity, only the ego vehicle interaction with the second vehicle is considered for cluster formation and visualization.

The parameters for varying scenarios are then reduced to the ego vehicle velocity v_1, and vehicle 2 velocity v_2, which can be further reduced by considering the ratio of velocities v_1/v_2. Using the same base path for the two vehicles from Fig. 1, the minimum distance and the angles (relative heading θ_{rel}, angle ϕ_{c} at the point of maximum criticality corresponding to vehicle 1 frame) as a function of the velocity ratio of the actors is shown in Fig. 4.

The scenario set for the vehicle paths consists of an infinite set of scenarios with velocity ratio v_1/v_2, assumed to be varying between 0 and 2, as a continuous parameter. To sample a subset from this infinite set, the velocity ratio is discretized with a 0.01 step size, thereby resulting in 200 points (excluding $v_1/v_2 = 0$) forming 200 scenarios, with each set of the two angles representing a scenario. The actors

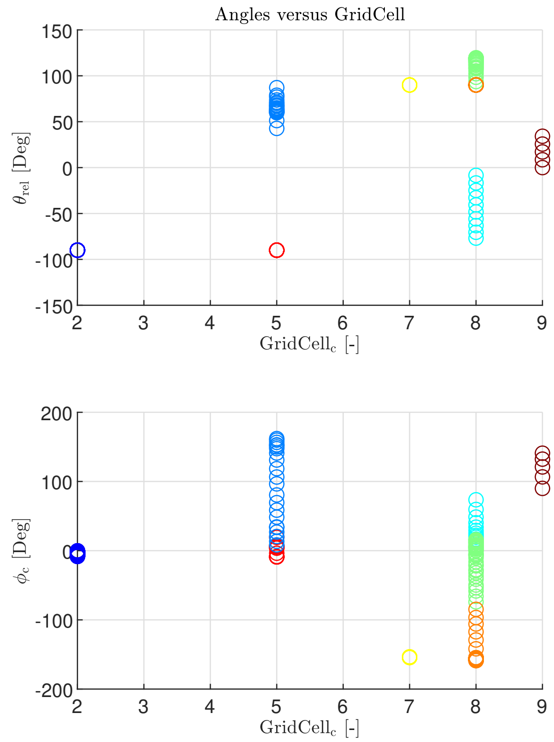

Fig. 3. Clustered 2D representation of angles, with respect to GridCell number GC_{c} dissimilarity metric. The different colors represent clusters based on the dissimilarity.

are modeled to start moving from their initial position at the start of the simulation time without any delays. From Fig. 4, there are two velocity ratio zones where the minimum distance is zero, which indicates a collision. The first zone occurs at $v_1/v_2 < 1$, which indicates that Vehicle 1 is slower than Vehicle 2. This can occur at the grid cell 5 in the road layout. The second zone at $v_1/v_2 > 1$ indicates the vice versa and can occur at grid cell 8 in the road layout. When the velocity ratio is 1, the two vehicles stay clear of each other in their respective paths as a near-miss condition.

According to the path defined for the test vehicle, the minimum distance scene can occur at the grid cell numbers $\{2, 5, 7, 8, 9\}$. As per the definition of dissimilarity, the occurrence of the minimum distance at a different grid cell between two scenarios results in 100% dissimilarity ($\mathfrak{d} = 1$). Hence there are five clusters with a dissimilarity of 1, based on 5 distinct grid cells. However, there can be more than five clusters depending on dissimilarity thresholds. For example, in Fig. 3, the points at grid cell 5 can be divided into at least two clusters: one at positive angles of θ_{rel} ($\approx$ 40° to 75°) and the other at negative angles ($\theta_{\mathrm{rel}} \approx -90°$).

To cluster the scenarios with the formulated dissimilarity semimetric, k-medoids clustering is applied. $k = 8$ is selected by evaluating the highest average silhouette score [18] for clustering values of k ranging between 5 and 15. The result of clustering the scenarios with $k = 8$ is shown

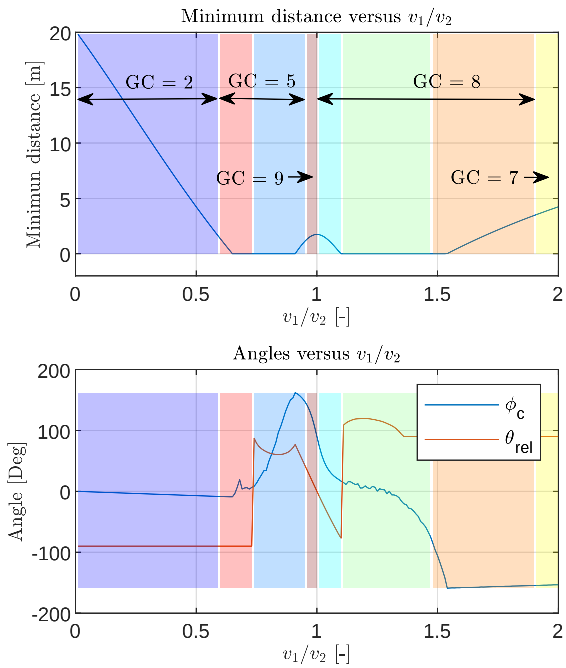

Fig. 4. Minimum distance and continuous variables ($\theta_{\mathrm{rel}}, \phi_{\mathrm{c}}$) as a function of velocity ratio of actors, with the found clusters of scenarios depicted by the different colored regions.

in Fig. 3, where the 8 colors represent the 8 clusters. It can be seen that the three extra clusters are formed now at grid cells 5 and 8. The split into two clusters at grid cell 5 is as expected. The split into three clusters at grid cell 8 is due to the variation in relative heading angle and angle in the local frame of the test vehicle.

To further enhance visualization, the cluster regions based on the varying velocity ratio between the actors are shown in Fig. 4. A 2D representation of the mean vehicle orientation at maximum criticality from each of the 8 scenario clusters is shown in Table. I. The clustering of scenarios with the dissimilarity semimetric indicates the possibility of selecting *diverse* scenarios, thus considerably reducing the number of scenarios from 200 to 8, in the example given. *Note: The dissimilarity semimetric is a novel approach for distinguishing scenarios by examining safety-critical scenes. This method differs from prior approaches [12]–[14], which assess dissimilarity using the entire scenario trajectory and therefore cannot be quantitatively compared.*

V. Conclusions

The main challenge this paper aims to tackle is ensuring diversity within a scenario generation framework for Automated Driving Systems (ADSs) by preventing the creation of redundant scenarios. Existing approaches in literature address this challenge by comparing scenarios using dissimilarity measures, which use complete trajectory information, or scenario parameter values. From a safety verification standpoint, however, it is more important to compare scenarios based on the most critical scenes in the scenario. To this end, we present a generic framework to formulate a dissimilarity (semi)metric based on the most critical scene, to compare driving scenarios accurately, thus preventing redundancies. The primary findings include:

1) Abstraction of the road network in an System of Interest (SOI) using mesh grids offers clear visualization of the scenario, allowing the inclusion of high-level trajectory information into the scenario comparison. The mesh grids are flexible and can be adapted to different resolutions, catering to various testing requirements.
2) The dissimilarity is determined based on the maximum criticality scene. This focuses the comparison on parts of the scenarios that are most relevant from a safety verification perspective and eliminates redundancies.
3) Clustering of scenarios using the dissimilarity metric indicates the possibility of selecting non-redundant scenarios for testing ADSs.

It is also essential to highlight research areas that can be further explored in this domain. Different measures can be explored for the selection of the most critical scene, instead of minimum distance, to study its effect on the dissimilarity metric. The current formulation of the dissimilarity metric can be extended to include more discrete and continuous features, for instance, criticality caused due to the occlusion (static and dynamic) of objects. The methodology employed, the framework developed, and the results presented in this paper establish a solid foundation for future investigations in these areas.

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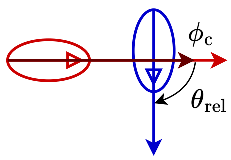
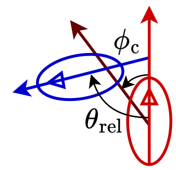
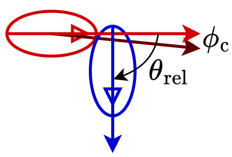
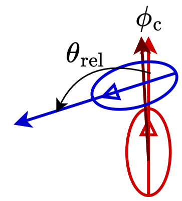
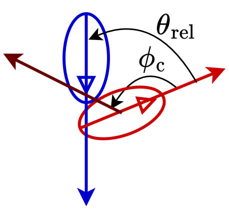
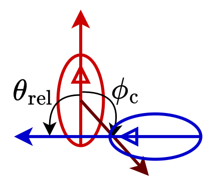
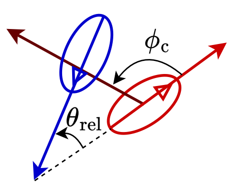
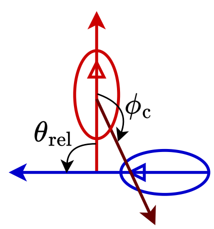
TABLE I

MEAN VEHICLE ORIENTATION AT THE INSTANT OF MAXIMUM CRITICALITY FROM THE 8 SCENARIO CLUSTERS

No.	**Vehicle Orientation**	**Values**	**No.**	**Vehicle Orientation**	**Values**
1		$\theta_{rel} = -90°$ $\phi_c = 0°$ ActorType$_c$ = 'Vehicle-Vehicle' GridCell$_c$ = 2	5		$\theta_{rel} \approx -75°$ $\phi_c \approx 30°$ ActorType$_c$ = 'Vehicle-Vehicle' GridCell$_c$ = 8
2		$\theta_{rel} = -90°$ $\phi_c \approx -10°$ ActorType$_c$ = 'Vehicle-Vehicle' GridCell$_c$ = 5	6		$\theta_{rel} \approx 110°$ $\phi_c \approx 0°$ ActorType$_c$ = 'Vehicle-Vehicle' GridCell$_c$ = 8
3		$\theta_{rel} \approx 75°$ $\phi_c \approx 130°$ ActorType$_c$ = 'Vehicle-Vehicle' GridCell$_c$ = 5	7		$\theta_{rel} = 90°$ $\phi_c - 90°$ ActorType$_c$ = 'Vehicle-Vehicle' GridCell$_c$ = 8
4		$\theta_{rel} \approx 30°$ $\phi_c \approx 110°$ ActorType$_c$ = 'Vehicle-Vehicle' GridCell$_c$ = 9	8		$\theta_{rel} = 90°$ $\phi_c \approx -160°$ ActorType$_c$ = 'Vehicle-Vehicle' GridCell$_c$ = 7

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